

Final Report

for

NASA NAGW-2848

"MHD Studies of the Magnetosheath and the Magnetopause"

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I. Introduction

This is the final report for the project NAGW-2848 entitled "MHD Studies of the Magnetosheath and the Magnetopause." It ran from April 1992 to December 1995. The research has been centered on the dynamics and structure of the magnetosheath and the magnetopause, a new theory of MHD shock waves, evolution of intermediate shocks, and shock waves in an anisotropic plasma.

II. Summary of Accomplishments

A. Dynamics of the magnetosheath and the magnetopause

We have been using MHD to study the global aspect of the dynamics at the magnetosheath. We used a time asymptotic method to obtain steady state solutions for the MHD flow past a sphere (Wu, 1992). The results indicate the formation of a depletion layer near the obstacle due to the increase of the magnetic field, as was predicted by Lees (1964). However, our results indicate that along the earth-sun line (which is also a stagnation streamline) the plasma density increases first and then decreases from the bow shock to the magnetopause. This shows that the flow after the shock is similar to those of gas dynamics. In contrast, Lees (1964) predicted that the density should decrease monotonically. The discrepancy can be traced back to the pressure balance assumption of Lees, which is shown in our calculation to be valid only near the obstacle but not valid in the region right after the shock. Our results also show that the depletion layer exists throughout the whole dayside magnetopause and is not restricted to the subsolar region. Our results show a nearly linear decrease of the normal velocity, a magnetic field pileup and a density drop on approach to the magnetopause. Phan et al. (1994) found that this is consistent with their observations for northward IMF cases.

When the IMF direction is tilted from the solar wind flow, the north-south symmetry of the magnetosheath flow is broken. When the Alfvén Mach number is small, we found that the effect of the magnetic field is so strong that it influences the shape and the position of the bow shock (Wu, 1993). Thus the bow shock location is a function of the IMF direction, the Alfvén Mach number, and the sonic Mach number. The model results agree with the recent Phobos-2 bow shock data at Mars (Slavin et al., 1992). During two subsolar bow shock traversals, Phobos-2 found one event with a strong ($M_S=5.9$, $M_A=5.1$) quasi-parallel ($\theta_{BN}=40-60^\circ$) bow shock at a distance of $1.5 R_M$ from the center of Mars. This is very near the mean position derived from a number of Mars missions. The second event yielded a much weaker ($M_S=10$, $M_A=1.8$) quasi-perpendicular ($\theta_{BN}=70^\circ$) bow shock at the unusual distance of $2.8 R_M$, or 3 to 4 times the average subsolar shock altitude. The model calculations show such strong dependence (Wu, 1993).

B. Intermediate shocks in dissipative MHD and Kinetic theory

It has been known for some 40 years that the MHD Rankine-Hugoniot relations have six shock solutions -- fast and slow shocks, and four different intermediate shocks. Application of the so-called MHD "evolutionary conditions" (Akhiezer et al., 1958; Taniuti, 1962; Kantrowitz and Petschek, 1966) soon ruled out the intermediate shocks.

A new theory of MHD shock waves has recently emerged from a series of

numerical and analytical investigations based on the dissipative MHD equations (Wu, 1987, 1988a, 1988b, 1990) and the Cohen-Kulsrud-Burgers (CKB) equation (Kennel et al., 1990; Wu and Kennel 1992a, 1992b). We found that intermediate shocks do form by wave steepening, and that the evolution and structure of intermediate shocks are related: in addition to the upstream and downstream states, specification of an intermediate shock's structure is required to determine its evolution. (Thus, the usual shock-capturing numerical schemes, which do not consider shock structures, cannot apply when there are intermediate shocks.)

We have also shown that there exists a new class of time-dependent shock-like structures, which do not obey Rankine-Hugoniot conditions since they violate coplanarity. These so-called "time-dependent intermediate shocks" play an essential role in the new MHD shock theory. On the one hand, they are the neighboring states of the regular coplanar intermediate shocks; on the other hand, they approach very broad rotational discontinuities at large time. In non-coplanar Riemann problems, time-dependent 2 - 3 intermediate shocks evolve in time as a localized self-similar structure whose strength decreases as $t^{-1/2}$, and whose width expands as $t^{1/2}$ (Wu and Kennel, 1992a). For the time-dependent 2 - 3 intermediate shocks, we have derived a set of structural relations, similar to Rankine-Hugoniot relations, between the plasma properties and the magnitude of the transverse magnetic field, which we hope can help identify them at the magnetopause, in the solar wind, and elsewhere in space (Wu and Kennel, 1992c).

We have shown that a rotational discontinuity is infinitely wide in dissipative MHD, or in other words, a rotational discontinuity cannot exist with a finite width. As a consequence, time-dependent intermediate shocks are needed in non-coplanar situations. We have also considered kinetic effects on intermediate shocks. The results indicate that both the two-fluid and hybrid calculations follow a similar development to the MHD case and show the formation of intermediate shocks (Wu and Hada, 1991a). Similarly, rotational discontinuities are shown to be unstable in both two-fluid and hybrid models (Wu and Hada, 1991b).

We expect intermediate shocks to be involved at the earth's magnetopause during magnetic reconnection process. Intermediate shocks may also exist in the solar wind. Indeed, Chao et al., (1993) has found candidates for intermediate shocks from Voyager 1 and 2 data. It is also suggested that intermediate shocks may constitute the Alfvén shock trains upstream of the earth's bow shock and in the interaction of the solar wind with Comet Giacobini-Zinner (Kennel et al., 1988). We have considered evolution of Alfvén waves subject to a long wavelength, plane-polarized, monochromatic instability (Malkov et al., 1991). The calculated waveforms resembled those observed.

The small amplitude MHD Riemann problem based on the CKB equations has been completed recently (Wu and Kennel, 1993). We found that the time-dependent 2 - 3 intermediate shock is important because it is the attractor of all non-coplanar Riemann systems at large time. Since there are literally hundreds of cases of the full MHD Riemann problem, we have limited our study to the constant-pressure Riemann problem, in which the upstream and downstream states have the same total pressure (Wu, 1995a). These solutions will be pertinent to sub-fast flows in the Earth's magnetic tail and near the magnetopause, two of the most important applications of intermediate shock theory. Although the Riemann solutions are complicated, several simple rules were found. When

the transverse magnetic fields of the upstream and downstream states point in opposite directions, an intermediate shock is formed. A condition is found regarding the position of the intermediate shock. When the transverse magnetic fields are in the same direction, no intermediate shock is involved if the velocity shear is small. However when the velocity shear is large, two intermediate shocks can form. We have also shown that for a high β plasma, as B_n (normal magnetic field component) becomes small, the shock-frame fluid velocity becomes Alfvénic for slow shocks, intermediate shocks and the front of a slow rarefaction wave, and that the Walén relation, which is exact for a rotational discontinuity, can also be well satisfied by them. Therefore observation of the Walén relation at the magnetopause does not necessarily mean the existence of a rotational discontinuity.

Since pressure anisotropy exists in the magnetosheath, at the magnetopause, and in the magnetotail, we have begun to consider shock waves and discontinuities in anisotropic plasmas (Wu, 1995b, 1996). We showed the existence of intermediate shocks, two types of contact discontinuities and rarefaction shocks. In particular, we showed that all four types of slow shocks in an anisotropic plasma can be formed through wave steepening. Across the four types of slow shocks, density can either increase or decrease and the strength of the magnetic field can also either increase or decrease. We also showed that the wave structure and the Riemann solution are very complicated and that the Riemann solution can be very different from that of MHD and thus can lead to a different physical picture, such as regarding the magnetic reconnection process. In an anisotropic plasma, the slow wave can move faster than the intermediate wave and thus it is possible for an intermediate shock to follow a slow shock. Indeed, recent observation of an anomalous slow shock and a rotational discontinuity (or intermediate shock) supports the results from the anisotropic plasma model (Walthour et al., 1994).

C. Development of central schemes for multi-dimensional magnetohydrodynamics

In last 15 years, upwind differencing schemes have been very popular for solving hyperbolic partial differential equations with discontinuous solutions. We have applied the upwind scheme to MHD (Brio and Wu, 1988), and since then several versions of upwind MHD schemes have been constructed (Zachary and Colella, 1992; Dai and Woodward, 1994a,b; Gombosi et al., 1994). Just as upwind schemes for gas dynamics, the upwind MHD schemes are very robust and capture shocks and discontinuities in very few grid points. However an upwind MHD scheme is very difficult to construct because it involves solving MHD Riemann problems. It is also very expensive to run since it requires many more operations and is difficult to vectorize. On a Cray computer, it can be two-order-of-magnitude increase in cpu time compared to a vectorized Lax-Wendroff code.

A family of new schemes that are non-oscillatory, second-order, and central difference for hyperbolic conservation laws has been constructed by Nessyahu and Tadmor (1990). The approximation can be viewed as a natural extension of the first-order Lax-Friedrichs scheme. In particular, TVD (total variation diminishing) is proved for the scalar case. Unlike the upwind scheme, no Riemann solver is required and thus the scheme is rather inexpensive and easy to construct. These new central schemes are very much as a two-step Lax-Wendroff scheme in its structure and their cpu time per grid update is therefore about the same as a two-step Lax-Wendroff scheme. However because of the

requirement for numerical stability the central schemes require the time-step size to be half of the usual CFL condition.

We have recently constructed second-order and third-order central schemes for MHD (Tadmor and Wu, 1996). The test results compare favorably with that obtained by the second-order upwind scheme (Brio and Wu, 1988). We have extended the second-order central scheme to 2D MHD and have successfully simulated the Kelvin-Helmholtz instability including the shock formation due to the vortex flow (Miura, 1984; Wu, 1986).

D. Other topics: Spherical implosion of shock waves

The structure and stability of shocks generated by spherical and cylindrical implosions have received much attention in the past decade. The impetus has come from astrophysics and plasma fusion research. More recently it has been conjectured that such shocks are an essential part of the mechanism responsible for sonoluminescence, i.e., the light which under certain conditions is emitted from a bubble of gas trapped in a liquid and compressed by incident spherically symmetric sound waves. In a series of papers (Wu and Roberts, 1993, 1994; Roberts and Wu, 1996), we have successfully modeled the sonoluminescence phenomenon and studied the structure and stability of a converging spherical shock wave.

III. References

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- Wu, C.C., Effects of solar wind conditions and interplanetary magnetic field on the bow shock and the magnetosheath flow, AGU Fall meeting in San Francisco, December, 1993.
- Wu, C. C., Magnetohydrodynamic Riemann Problem and the Structure of the Magnetic Reconnection Layer, *J. Geophys. Res.*, 100, 5579, 1995a.
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IV. Publications and talks resulting from the support

A. Publications

- C.C. Wu and C.F. Kennel, Structural relations of time-dependent intermediate shocks, *Geophys. Res. Letts.*, 19, 2087, 1992.
- C.C. Wu and C.F. Kennel, The small amplitude MHD Riemann problem, *Phys. Fluids*, B5, 2877, 1993.
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- C.C. Wu, Intermediate shocks, *Physica Scripta*, T60, 97, 1995.
- P.H. Roberts and C.C. Wu, Structure and stability of a spherical implosion, *Physics Letters A*, in press, 1996.
- C.C. Wu, Shock waves in an anisotropic plasma, *J. Geophys. Res.*, to be submitted, 1996.
- E. Tadmor and C.C. Wu, Central scheme for the multidimensional MHD equations, *J. Computational Phys.*, to be submitted, 1996.

B. Talks presented at meetings

- C.C. Wu, Structure and evolution of intermediate shocks, AGU meeting in Hong Kong, August, 1992. (Invited)
- C.C. Wu and C.F. Kennel, Structural relations for time-dependent intermediate shocks, AGU Fall meeting in San Francisco, December, 1992.
- C.C. Wu, Intermediate shocks, IUGG meeting in Argentina, August, 1993.(Invited)
- C.C. Wu, Effects of solar wind conditions and interplanetary magneticfield on the bow shock and the magnetosheath flow, AGU Fall meeting in San Francisco, December, 1993.
- C.C. Wu, The MHD Riemann problem and the structure of the reconnection layer, Chapman Conference on Physics of the Magnetopause, SanDiego, March, 1994.
- C.C. Wu, A new theory of MHD shock waves, Kyoto Workshop on Nonlinear Waves, Kyoto, Japan, June, 1994. (Invited)
- C.C. Wu, Shock waves in an anisotropic plasma, AGU Fall meeting in San Francisco, December, 1994.
- C.C. Wu, Shock waves in anisotropic plasmas, AGU Fall meeting in San Francisco, December, 1995.